Calibration of Differential Phase Map Compensation Using Single Axis Rotation

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BIOGRAPHY

Eric Sutton received a bachelor’s degree in Electrical Engineering in 1985 and a master’s degree in 1988, both from Southern Illinois University. He worked for Johns Hopkins Applied Physics Laboratory from 1988 to 1992 in the Ocean Data Acquisition Program, where he did work in image enhancement and processing of side scan sonar data. Mr. Sutton received a doctorate in Electrical Engineering from the University of Iowa in 1996, where the primary focus of his research was in time varying ionospheric tomography. Mr. Sutton has recently joined Rockwell Collins in Cedar Rapids, Iowa and is doing research in GPS attitude determination systems.

ABSTRACT

This paper describes a new technique for measuring the differential phase map between two GPS antennas. The primary application for a differential phase map is to increase the accuracy of GPS attitude determination systems. The technique proposed in this paper uses a rotating test fixture to increase phase map coverage, assist in accurately determining true antenna positions, and reduce the effect of environmentally generated multipath errors. Use of the rotating test fixture also simplifies integer ambiguity resolution. The proposed technique is less expensive than anechoic chamber testing, and less time consuming than static testing. Although at this time data collection has been limited, several examples of phase maps generated using the new technique will be presented, and in one case a phase map using the new technique will be compared to a phase map generated by anechoic chamber testing.

1 INTRODUCTION

GPS attitude determination systems use the difference in phase measurements between two or more antennas to determine the attitude of a host vehicle. The phase differences measured between two antennas exhibit errors that are a function of the angle of arrival of the satellite signal with respect to the antennas. These errors can be minimized through careful design of antenna mounting arrangements [8], and the overall accuracy of the system can be increased by increasing the distance between the antennas; however, in many applications nonideal antenna mounting arrangements must be used, and the distance between the antennas is limited. Under these conditions, the differential phase measurements can contain errors that are over a centimeter in magnitude, which translates into significant errors in attitude. Phase map compensation is not difficult to implement in software, and its effectiveness has been well documented [3][4][5][9]; however,
the field testing or anechoic chamber testing required to measure the phase map can be expensive and time consuming. This paper will present a simple and inexpensive method to calculate the differential phase map using a single degree of freedom rate table and live satellite signals.

The phase center of an antenna is a point on or near the surface of the antenna to which all phase measurements are referenced. For an ideal antenna, it is assumed that the antenna can be rotated around its phase center without affecting the phase measurements. However, for a real antenna, the position of the phase center varies depending on the angle of arrival of the incoming electromagnetic waves. Variations in the phase center can be compensated by assuming a fixed average phase center, then modeling a varying bias in the phase measurements, where the bias is a function of the angle of arrival. The antenna phase map is a polar plot of the measurement bias as a function of azimuth and elevation of the satellite with respect to the antenna array.

It is possible to calculate a phase map for a single antenna using a two degree of freedom test fixture and live satellite signals [10]. However, for a differential phase map between two antennas for use in an attitude determination system, the orientation of the test fixture must be very precisely known.

This paper will present a method of calculating the antenna phase map by placing the antenna array on a test fixture that revolves about a single fixed axis at a constant rate. As the test fixture rotates, the line of sight to each satellite traces a spiral through the phase reception map, as illustrated for a single satellite in Figure 1. The effect of multipath errors is reduced, since each point in the phase reception map represents an average from several satellites at different azimuth angles. The integer cycle ambiguity is solved without an ambiguity search by using the known rotation rate of the test fixture. In addition, coverage of the phase map can be quite good for relatively short data collection times.

No a priori knowledge of the orientation of the test fixture is required. Since the problem of calculating the orientation of a rotating antenna array is equivalent to the problem of calculating the orientation of a fixed antenna array with a rotating satellite constellation, the orientation of the antenna array can be
determined very precisely from the phase measurements, so the test fixture is essentially self-calibrating. The singular value decomposition is used to optimally calculate the axis of rotation and to rotate from local level coordinates into a geometrically convenient coordinate system. The rate at which the rate table revolves should be approximately known, but small variations and drifts in the rate can be compensated. Receiver differential clock drift is modeled to reduce vertical errors.

Algorithms for real time attitude determination of a spinning platform have been proposed [1][2][7]; however, the technique presented in this paper is an optimal batch processing algorithm under the assumption that the axis of rotation is constant. The optimal algorithm is used because the phase map is very sensitive to errors in the estimate of the position of the antennas.

This paper will also present an error analysis of the resulting phase map and data to show that this method of calculating a phase map is consistent with itself and with anechoic chamber data.

2 INTEGER CYCLE AMBIGUITY RESOLUTION USING 180° ROTATION

One of the advantages of rotating the antennas at a constant rate is that the calculation of the integer cycle ambiguity vector is simplified and any uncertainty in the validity of the integer ambiguity solution is essentially removed. The method presented here relies on 180° rotation of the baseline vector; however, the technique may be adapted so that rotation through any known angle may be used.

The unit line of sight matrix is given by

\[ L = \begin{bmatrix} l_1 & l_2 & \cdots & l_M \end{bmatrix}, \]

(1)

where each column vector \( l_m \) is a unit line of sight vector to the \( m \)th satellite in local level coordinates. The measurement equation at time \( t_i \) is given by

\[ \phi_i = L_i^T d_i + 1 \delta t_i + N, \]

(2)

where \( \phi_i \) is a vector of single difference phase measurements, \( L_i \) is the unit line of sight matrix, \( d_i \) is the baseline vector, \( 1 \) is a vector containing ones, \( \delta t_i \) is the clock offset between receivers, and \( N \) is a vector of integer ambiguities. The units of equation (2) are wavelengths, and the coordinate system for both \( L_i \) and \( d_i \) is local level. The subscript notation is used to indicate the epoch, i.e. \( \phi_i \) is shorthand for \( \phi(t_i) \). Later in this paper the subscript will be omitted whenever it is clear that only one epoch is being considered.

Suppose we have an a priori estimate of the rate at which the rate table is turning. Let time be chosen so that the rate table has turned 180° between \( t_i \) and \( t_j \). The measurement equation at time \( t_j \) is given by

\[ \phi_j = L_j^T d_j + 1 \delta t_j + N. \]

(3)

If only satellites that are continuously tracked from \( t_i \) to \( t_j \) are included in (2) and (3), then the vector of integer ambiguities \( N \) is the same for both equations. The integer ambiguities can be eliminated by subtracting (3) from (2):

\[ \phi_i - \phi_j = L_i^T d_i - L_j^T d_j + 1 \delta t_i - 1 \delta t_j \]

(4)
Since the rate table has turned 180° between \( t_i \) and \( t_j \), the two baseline vectors point in exactly opposite directions:

\[
d_j = -d_i
\]

Define the time differenced clock offset and phase measurement vector:

\[
\Delta \delta t = \delta t_i - \delta t_j \quad (6)
\]
\[
\Delta \phi = \phi_i - \phi_j \quad (7)
\]

After applying the substitutions given by (5), (6), and (7), the time differenced measurement equation becomes

\[
\Delta \phi = (L_i^T + L_j^T) d_i + 1 \Delta \delta t. \quad (8)
\]

Since the clock offset is common to all satellites, it can be eliminated by subtracting a reference satellite from all the other satellites. The triple differenced measurement equation is given by

\[
\nabla \Delta \phi = \nabla (L_i^T + L_j^T) d_i. \quad (9)
\]

Define the matrix \( H \) as follows:

\[
H = \nabla (L_i^T + L_j^T) \quad (10)
\]

Then, the unweighted least squares solution for the baseline vector is given by

\[
d_i = (H^T H)^{-1} H^T \nabla \Delta \phi. \quad (11)
\]

For subsequent processing we will need the integer ambiguity vector and the clock offset for at least one epoch. The clock offset can be eliminated from equation (2) by subtracting a reference satellite from all the other satellites:

\[
\nabla \phi_i = \nabla L_i^T d_i + \nabla N \quad (12)
\]

Then the double differenced integer ambiguity vector is given by

\[
\nabla N = \text{round} \left( \nabla \phi_i - \nabla L_i^T d_i \right). \quad (13)
\]

The single differenced integer ambiguity vector can be constructed from the double differenced integer ambiguity vector simply by appending a zero at the position of the reference satellite. For example, if the first satellite is chosen as the reference satellite, then the single difference integer ambiguity vector is given by

\[
N = \begin{bmatrix} 0 \\ \nabla N \end{bmatrix}. \quad (14)
\]
Figure 2: Modeling of clock drift using a polynomial curve fit. The lighter color shows the single epoch estimates, and the dark curve shows the overall estimate.

The integer ambiguity for the reference satellite is common to the single difference integer ambiguities for all the satellites, so the integer ambiguity for the reference satellite can be absorbed into the clock offset. The clock offset can be estimated using

$$\delta t_i = \frac{1}{M} \mathbf{1}^T (\phi_i - L_i^T d_i - N).$$  \hspace{1cm} (15)$$

There can be some difficulty with the estimated clock offset changing by a integer multiple of the wavelength when the reference satellite changes; however, this can be avoided by propagating the clock offset between epochs, then solving for the ambiguity using single differenced instead of double differenced measurements.

3 SINGLE EPOCH LEAST SQUARES SOLUTION

Once the integer ambiguity vector is found for at least one epoch, the solution can be propagated forward and backward in time. The method used here assumes that movement of the baseline vector between epochs combined with drift of the clock offset between epochs is much less than a half of a wavelength. This is usually a very easy requirement to satisfy.

The single difference integer ambiguity vector is calculated from

$$N = \text{round} \left( \phi - L^T d^{(-)} - \delta t^{(-)} \right),$$ \hspace{1cm} (16)$$

where \(d^{(-)}\) and \(\delta t^{(-)}\) are values from either the next or the previous epoch. Define the matrix \(G\) as follows:

$$G = \begin{bmatrix} L^T & 1 \end{bmatrix}$$ \hspace{1cm} (17)$$

Then the least squares solution for the baseline vector and the clock offset is given by
Figure 3: Improvement in vertical accuracy after modeling clock drift. The lighter color shows the measurements before modeling clock drift, and the darker color shows the measurements after modeling clock drift.

\[
\frac{d}{dt} = (G^T G)^{-1} G^T (\phi - N).
\]

(18)

Although the integer ambiguity vector is propagated between epochs, the baseline solution and clock offset for each epoch is calculated using information from only a single epoch.

4 CALCULATE CLOCK ERRORS

Using the integer ambiguity vector and estimate of the baseline previously calculated, the clock offset for each epoch is given by

\[
\delta t = \frac{1}{S} L^T (\phi - L^T \hat{d} - N),
\]

(19)

where \( S \) is the number of satellites. The estimate of the clock offset \( \hat{\delta t} \) can then be calculated by doing a curve fit to the set of points \((t_m, \delta t_m)\), as illustrated in Figure 2.

Modeling the clock drift dramatically increases the accuracy of the estimate of the vertical component of the baseline, as illustrated in Figure 3. The up component of the baseline vector contains a sinusoidal component, because the test fixture is not exactly level. The up component departs from a pure sine wave due to the phase jump errors, in addition to other errors.

The solution for the components of the baseline vector at this point in the processing is shown in Figure 4. The east and north components trace sinusoids with amplitude approximately equal to the baseline length. The up component contains a small sinusoidal component. The next step is to fit the data of Figure 4 to the model of baseline motion. This will be covered in the following section.
5 ALL EPOCH SOLUTION USING SINGULAR VALUE DECOMPOSITION

The final phase map solution is very sensitive to errors in the estimate of the baseline position. Therefore, it is necessary to base the solution for the baseline vector at any single point in time on all of the data collected. This process is conceptually a sine wave curve fit; however, in practice, the data is transformed so that a linear curve fit can be used.

The axis of rotation of the rate table is approximately orthogonal to the baseline vector for every epoch. If \( d_m \) is the baseline vector at time \( t_m \), and \( r \) is the axis of rotation, then for all \( m \) we want

\[
d_m^T r = 0. \tag{20}
\]

The requirement of equation (20) can be written in matrix notation. Define the matrix \( D \) as follows:

Each row of \( D \) is a normalized baseline vector. The baseline vectors are normalized so that errors in the length of the estimate of the baseline will not interfere with the estimate of the axis of rotation. In matrix notation, (20) is equivalent to

\[
D r = 0, \tag{21}
\]

In other words, the null space of \( D \) has dimension of 1, and \( r \) is the basis vector for the null space of \( D \). Equations (20) and (21) are written for noiseless data; however, in reality the estimated baseline vectors contain noise, so (21) cannot be exact. Instead, we must solve the following constrained optimization:

\[
\min ||D r|| \text{ subject to } ||r|| = 1 \tag{22}
\]

The above optimization problem can be solved using the singular value decomposition (SVD). Let the SVD of \( D \) be given by

\[
D = U \Sigma V^T, \tag{23}
\]

where \( U \) and \( V \) are orthonormal matrices, and \( \Sigma \) is a diagonal matrix. Furthermore, let

\[
r' = V^T r. \tag{24}
\]

The matrix \( \Sigma \) can be written as

\[
\Sigma = \begin{bmatrix}
\sigma_1^2 & 0 & 0 \\
0 & \sigma_2^2 & 0 \\
0 & 0 & \sigma_3^2 \\
0_{(M-3) \times 3}
\end{bmatrix}, \tag{25}
\]

where the singular values satisfy the inequality \( \sigma_1^2 \leq \sigma_2^2 \leq \sigma_3^2 \). Also, the matrix \( V \) can be written as

\[
V = \begin{bmatrix}
v_1 \\
v_2 \\
v_3
\end{bmatrix}, \tag{26}
\]
where \(v_1, v_2, \) and \(v_3\) are column vectors. Then, the quantity to be minimized can be expressed as

\[
\|Dr\| = \|U\Sigma V^T r\| \\
= \|\Sigma V^T r\| \\
= \|\Sigma r'\| \\
= \sqrt{\sigma_1^2 (r')_1^2 + \sigma_2^2 (r')_2^2 + \sigma_3^2 (r')_3^2}.
\]

The minimum value of (27) is \(\sigma_3\) and the value of \(r'\) at the minimum is \(\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T\). Therefore,

\[
r = Vr' = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = v_3.
\]

In other words, the axis of rotation is given by the third column of \(V\), so \(V\) can be written as

\[
V = \begin{bmatrix} v_1 & v_2 & \pm \hat{\varphi} \end{bmatrix}.
\]

There is a sign ambiguity in the calculation of \(r\), i.e. if the axis of rotation is approximately vertical, then \(v_3\) can point either up or down. We will assume that the sign of \(r\) is determined so that it always points up; otherwise, there may be an unintended reflection of coordinates when calculating the phase map.

The matrix \(V\) also represents a rotation in physical space, and the third column of \(V\) projects any given vector onto the axis of rotation. In other words, \(V\) represents a rotation of coordinates where the third coordinate axis is aligned with the axis of rotation. Define \(D'\) to be the matrix of baseline vectors rotated by \(V\):

\[
D' = DV = U \Sigma
\]
The matrix $V$ may also contain other reflections and rotations; however, these will cancel out when the inverse rotation represented by $V^T$ is applied. Label the columns of $D'$ as follows:

$$D' = [ x' \quad y' \quad z' ]$$

(31)

In the primed coordinate system, the rows of $D'$ are vectors that rotate at a constant rate in the $x$-$y$ plane. Calculate the rotation angle from an arbitrary starting point as follows:

$$\theta_m = \arctan \frac{y'_m}{x'_m}$$

(32)

The estimate of the true rotation angle denoted by $\hat{\theta}_m$ is calculated by doing a curve fit to the set of points given by $(t_m, \theta_m)$ for all $m$. Note that it is necessary to unwrap the phase, since (32) will give the phase modulo $2\pi$. A linear curve fit is appropriate when the rotation rate is known to be absolutely constant. A second order polynomial fit can be used if there is a constant drift in the rotation rate, and a higher order curve fit can be used if necessary.

The estimate of the three components of the unit baseline vectors in the primed coordinate system is given by

$$\hat{x}_m' = \cos \hat{\theta}_m,$$

(33)

$$\hat{y}_m' = \sin \hat{\theta}_m,$$

(34)

$$\hat{z}_m' = 0,$$

(35)

The a priori model of the baseline movement is now incorporated into the estimate of the baseline vectors. Define $\hat{D}'$, the estimate of $D'$, as follows:

$$\hat{D}' = [ \hat{x}' \quad \hat{y}' \quad \hat{z}' ]$$

(36)

Then rotate the unit baseline vectors back to the local level coordinate system:

$$\hat{D} = \hat{D}' V^T$$

(37)

The rows of $\hat{D}$ are unit vectors. An estimate of the length of the baseline vector $\|\hat{d}\|$ can be calculated by averaging the lengths of the measured baseline vectors $d_m$ for all $m$. Let the estimate of the baseline vectors be denoted by $\hat{d}_m$; then the estimated baseline vectors are the rows of the matrix $\hat{D}$ scaled by $\|\hat{d}\|:

$$\begin{bmatrix}
\hat{d}_1^T \\
\hat{d}_2^T \\
\vdots \\
\hat{d}_M^T
\end{bmatrix}
= \|\hat{d}\| \hat{D}$$

(38)

An improved estimate of the baseline vectors can be calculated using the new estimate of the clock drift in the algorithm of the previous section. In theory, this is an iterative algorithm, and additional iterations could be performed, but in practice two iterations are sufficient.
6 CALCULATE THE PHASE MAP

The phase map consists of a plot of the phase residuals as a function of azimuth and elevation in a coordinate system that rotates with the antennas. Using previously calculated estimates of the baseline vector, clock offset, and integer ambiguity vector, the phase residual is calculated using

\[
\phi_r = \phi - L^T \hat{d} - 1\hat{\delta} - N. 
\]

A rotation matrix to transform local level coordinates into a coordinate system fixed to the antennas must be constructed. Let this matrix be denoted by \( T \). Suppose we adopt the convention that the \( x \) axis of the rotating coordinate system is aligned with the baseline vector, the \( z \) axis is aligned with the axis of rotation, and the \( y \) axis is determined so that the coordinate system is right handed; then the rotation matrix \( T \) must satisfy

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} = T \begin{bmatrix}
\frac{\hat{d}}{\|\hat{d}\|} \\
\frac{\hat{r}}{\|\hat{r}\|} \\
\hat{r} 
\end{bmatrix}.
\]

(40)

Since the matrix on the left is an identity matrix, and the matrix on the right is orthonormal, the rotation matrix is given by

\[
T = \begin{bmatrix}
\frac{\hat{d}}{\|\hat{d}\|} & \hat{r} \times \frac{\hat{d}}{\|\hat{d}\|} & \hat{r} 
\end{bmatrix}^T.
\]

(41)

The unit line of sight vectors are transformed into the rotating coordinate system using the rotation matrix \( T \):

\[
L' = TL
\]

(42)

The phase map is a function of the angle of arrival of the incoming electromagnetic wave with respect to the antenna array. The angle of arrival is given by the unit line of sight vector in the primed coordinate system; however, since the unit line of sight vector has three components but only two degrees of freedom, the unit line of sight vector is a poor way to parameterize the phase map. One way to parameterize the phase map is to use the heading and elevation of the unit line of sight vector in the primed coordinate system. If \( L' \) is a unit line of sight vector in the primed coordinate system, then the heading \( \psi' \) and elevation \( \theta' \) are given by

\[
\psi' = \arctan \frac{\mu_2}{\mu_1}, 
\]

(43)

\[
\theta' = \arcsin \frac{\mu_3}{\|\mu\|}.
\]

(44)

The heading and elevation can then be remapped into rectangular coordinates as follows:

\[
x' = \left( \frac{\pi}{2} - \theta' \right) \cos \psi'
\]

(45)
$$y' = \left( \frac{\pi}{2} - \theta' \right) \sin \psi'$$

Note that this primed coordinate system is not the same as the primed coordinate system in the section on the all epoch solution. The \((x',y')\) coordinate system is convenient for plotting the phase map. The phase map is divided into pixels in the primed coordinate system. All of the phase residuals that fall within each pixel are averaged to obtain the phase map.

### 7 PRELIMINARY RESULTS

Due to problems with data collection, it is not yet possible to present a complete error analysis using real data. However, with the limited data available at this time, it is possible to demonstrate that:

- Consistent results are obtained using data collected at different times.
- The phase map calculated using the technique presented in this paper is consistent with the phase map calculated from anechoic chamber data.

Two different antenna configurations were used; they will be designated configuration A and configuration B. The rotation rate was 1 degree per second, and the data collection rate was 1 sample per second.

Figure 5 shows a phase map using data from antenna configuration A starting at GPS time 149008 seconds and continuing for 68 minutes, and Figure 6 shows a phase map using data from antenna configuration A starting at GPS time 154525 seconds and continuing for 79 minutes. Both data sets are much shorter than the optimum length. The phase maps are plotted using the primed coordinate system given in the preceding section. The zenith is in the center of the plot, and the horizon is along the circular boundary of the region containing the phase map. The baseline vector points toward the top of the plot. The phase maps shown in Figure 5 and Figure 6 are highly correlated, showing that the technique presented in this paper produces consistent results.

Figure 7 shows a phase map for configuration B using anechoic chamber data, and Figure 8 shows a phase map for configuration B using the technique presented in this paper. The duration of data collection for Figure 8 is 176 minutes. The phase maps of Figures 3 and 4 are clearly correlated. The phase map from the anechoic chamber has much higher resolution, and this is the price that is paid to avoid anechoic chamber testing. On the other hand, it is often not possible to use such a high resolution phase map in a real time system.

### 8 CONCLUSIONS

This paper has presented a new technique for calculating differential phase maps. Differential phase maps are important for accurate attitude determination systems. The technique presented in this paper has the following advantages:

- Only the signals from live satellites are required.
- Only the phase measurements and line of sight vectors need to be measured and recorded.
Only rotation about a single axis at a reasonably constant rate is required.

The test setup is self-calibrating; no special alignment procedure is required.

The new technique is usually easier and much less expensive than anechoic chamber testing. The phase map calculated from anechoic chamber testing is expected to be more accurate; however, once the antenna reception pattern is compensated to within 1-2 mm, other error sources become much more significant for attitude determination systems.

Although preliminary results show that the new technique produces consistent results, future work should focus on thorough theoretical and practical error analysis of the new technique.

Rotating antenna arrays have advantages over fixed antenna arrays that could be useful in other applications. The integer cycle ambiguity problem is easier to solve with a rotating antenna array, and there is potential for reducing multipath errors using the spatial sampling provided by a rotating array. A rotating array could provide a very accurate attitude reference for applications where long baselines cannot be used.

REFERENCES


Figure 5: Phase map from rate table data for configuration A starting at GPS time 149008 sec.
Figure 6: Phase map from rate table data for configuration A starting at GPS time 154525 sec.
Figure 7: Phase map from anechoic chamber data for configuration B.
Figure 8: Phase map from rate table data for configuration B.