Homework 1

Work problems 2.5, 2.15, 2.19, and 3.1 from the book.

Homework 2

Work problems 3.4 and 3.10 from the book.

Additional problem 1:

The average and variance of all the pixels in $f(x, y)$ can be calculated as follows:

$$
\mu = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)
$$

$$
\sigma^2 = \frac{1}{MN - 1} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (f(x, y) - \mu)^2
$$

Let $h(n)$ be the histogram of the 8-bit image, $f(x, y)$, so the index $n$ ranges from 0 to 255, and $h(n_1)$ is the number of pixels in $f(x, y)$ with value $n_1$. Find expressions for calculating the mean and variance of the image using only the histogram, $h(n)$.

Additional problem 2:

Calculate the result of applying the mask on the left to the image data on the right:
Assume that the linear convolution is required; i.e. the result will be larger than the original image.

**Additional problem 3:**

Calculate the result of applying a $3 \times 3$ median filter to the image data below:

\[
\begin{array}{ccc}
3 & 3 & 4 \\
2 & 1 & 4 \\
4 & 1 & 0
\end{array}
\]

Extrapolate pixel values outside the image by copying from the nearest image pixel.

**Homework 3**


**Homework 4**

Work problems 4.9, 4.14, and 4.17 from the book.

**Homework 5**

Work problems 4.10, 4.20, and 4.21 from the book.

**Additional problem 1:**

**Part a**

Find an expression for the one-dimensional $M$-point DFT, $H(u)$, for the following one-dimensional mask:

\[
\begin{array}{ccc}
1 & 1 & 1
\end{array}
\]

HINT: $H(u)$ will be a real-valued function.
Part b

Use the results from Part a to find an expression for the $M \times N$ DFT, $H(u,v)$, of the $3 \times 3$ box filter:

$$
\frac{1}{9} \times \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{bmatrix}
$$

Homework 6

Work problems 5.10, 5.13, and 5.15 from the book.

Homework 7

Work problems 5.25 and 5.28 from the book.

Additional problem 1:

For the system

$$
g(t) = h(t) * f(t),
$$

show that

$$
S_{fg}(u) = H^*(u) S_{ff}(u)
$$

Additional problem 2:

Consider the two-dimensional circular $4 \times 4$ convolution:

$$
g(x, y) = h(x, y) \circledast f(x, y)
$$

This can be written as a matrix multiplication:

$$
g = H f
$$

where

$$
f = \begin{bmatrix}
 f(0,0) \\
 f(0,1) \\
 f(0,2) \\
 f(0,3) \\
 f(1,0) \\
 f(1,1) \\
 f(1,2) \\
 \vdots \\
 f(3,2) \\
 f(3,3)
\end{bmatrix}
$$

and

$$
g = \begin{bmatrix}
 g(0,0) \\
 g(0,1) \\
 g(0,2) \\
 g(0,3) \\
 g(1,0) \\
 g(1,1) \\
 g(1,2) \\
 \vdots \\
 g(3,2) \\
 g(3,3)
\end{bmatrix}
$$

and $h(x, y)$ is a $3 \times 3$ box filter. Find the $16 \times 16$ matrix $H$. 

3
HINT: H is of the following doubly block circulant form:

\[ H = \begin{bmatrix} H_0 & H_1 & H_2 & H_3 \\ H_3 & H_0 & H_1 & H_2 \\ H_2 & H_3 & H_0 & H_1 \\ H_1 & H_2 & H_3 & H_0 \end{bmatrix} \]

**Homework 8**

Work problems 6.5, 6.6, 6.12, 9.4, 9.14, and 9.27 from the book.

For problems 6.6 and 6.12, it is sufficient to fill in a table of the following form:

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<th>Color</th>
<th>R</th>
<th>G</th>
<th>B</th>
<th>H</th>
<th>S</th>
<th>I</th>
</tr>
</thead>
<tbody>
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<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
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<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
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<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Green</td>
<td>*</td>
<td>*</td>
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<td>*</td>
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</tr>
<tr>
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<td>*</td>
<td>*</td>
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<tr>
<td>Blue</td>
<td>*</td>
<td>*</td>
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<td>*</td>
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</tr>
<tr>
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<td>*</td>
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<tr>
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<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
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<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

**Additional problem 1:**

1. Find the dilation, erosion, opening, and closing of set \( A \) using structuring element \( B_1 \).

2. Find the dilation, erosion, opening, and closing of set \( A \) using structuring element \( B_2 \).

Working this problem on graph paper is recommended.

**Homework 9**

Work problems 10.17, 11.6 (parts b and c), 11.11, 11.14 from the book.
Homework 10

Work problems 8.3, 8.8, 8.13, 8.17, and 8.20 from the book.

For problem 8.8, use the following value for $Q$ instead of the value given in the book:

$$Q = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

For problem 8.20, assume that the autocorrelation function is

$$E[f_n f_{n-i}] = \sigma^2 \rho^i.$$  

Additional problem 1:

The one-dimensional DCT is given by:

$$C(u) = \sum_{n=0}^{N-1} 2x(n) \cos \left( \frac{(2n+1)u\pi}{2N} \right)$$

Define $y(n)$ as follows:

$$y(n) = \begin{cases} x(n) & \text{for } 0 \leq n \leq N - 1 \\ x(2N - 1 - n) & \text{for } N \leq n \leq 2N - 1 \end{cases}$$

Show that the $2N$-point DFT of $y(n)$ can be expressed as

$$Y(u) = W_{2N}^{-u/2} \sum_{n=0}^{N-1} 2x(n) \cos \left( \frac{(2n+1)u\pi}{2N} \right).$$

Recall that $W_{2N} = e^{-j\frac{2\pi}{2N}}$.

Use your results to propose an algorithm for calculating an $N$-point DCT using a $2N$-point DFT.