The forward and reverse transforms:

$$T(u) = \sum_{x=0}^{3} f(x) g(x, u)$$

$$f(x) = \sum_{u=0}^{3} T(u) h(x, u)$$

The $g$'s (and also the $h$'s) must satisfy an orthogonality condition:

$$\sum_{x=0}^{3} g(x, u_1) g(x, u_2) = \begin{cases} 
1 & \text{if } u_1 = u_2; \\
0 & \text{otherwise.}
\end{cases}$$
Confirm orthogonality for \( u_1 = 1 \):

\[
\sum_{x=0}^{3} g(x, 1) g(x, 0) = (1/2)(1/2) + (1/2)(1/2) + (-1/2)(1/2) + (-1/2)(1/2) = 0
\]

\[
\sum_{x=0}^{3} g(x, 1) g(x, 1) = (1/2)(1/2) + (1/2)(1/2) + (-1/2)(-1/2) + (-1/2)(-1/2) = 1
\]

\[
\sum_{x=0}^{3} g(x, 1) g(x, 2) = (1/2)(1/2) + (1/2)(-1/2) + (-1/2)(-1/2) + (-1/2)(1/2) = 0
\]

\[
\sum_{x=0}^{3} g(x, 1) g(x, 3) = (1/2)(1/2) + (1/2)(-1/2) + (-1/2)(1/2) + (-1/2)(-1/2) = 0
\]

Find the WHT of \( f(x) \):

\[
T(0) = \sum_{x=0}^{3} f(x) g(x, 0) = (-1)(1/2) + (0)(1/2) + (2)(1/2) + (3)(1/2) = 2
\]

\[
T(1) = \sum_{x=0}^{3} f(x) g(x, 1) = (-1)(1/2) + (0)(1/2) + (2)(-1/2) + (3)(-1/2) = -3
\]

\[
T(2) = \sum_{x=0}^{3} f(x) g(x, 2) = (-1)(1/2) + (0)(-1/2) + (2)(-1/2) + (3)(1/2) = 0
\]

\[
T(3) = \sum_{x=0}^{3} f(x) g(x, 3) = (-1)(1/2) + (0)(-1/2) + (2)(1/2) + (3)(-1/2) = -1
\]

This means that \( f(x) \) can be expressed as a weighted sum of the basis functions \( h(x, u) \):

\[
f(x) = (2)h(x, 0) + (-3)h(x, 1) + (0)h(x, 2) + (-1)h(x, 3)
\]
We could have found the WHT of $f(x)$ using matrix multiplication:

\[
\begin{bmatrix}
2 \\
-3 \\
0 \\
-1
\end{bmatrix} = \frac{1}{2}
\begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
-1 \\
0 \\
2 \\
3
\end{bmatrix}
\]

Notice that the matrix $A$ is symmetric; i.e. $A = A^T$.

We can confirm the orthogonality of all the basis functions at once by showing that $A^TA = I$:

\[
\begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
4 & 1 & -1 & -1 \\
1 & -1 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

At the same time, this shows that $A$ is its own inverse; i.e. $A = A^{-1}$.