Exam 2 Review

- One hour to complete
- Closed book, closed notes, no calculator necessary
- Two 8.5 by 11 sheets with anything you want on it:
  - You might want to make one sheet the one from the first exam
  - You will be allowed three sheets for the final
- Covers chapters 4, 5, 6, 9, 10, 11. (Chapter 8 will be on the final)
- Six questions

Chapter 4: Enhancement in the Frequency Domain

- Properties of the DFT
  - Symmetry properties
  - Periodicity
  - Centered & uncentered, frequency shifting
  - Spatial shifting
  - Scaling
  - Reciprocal relationship with spatial domain
- Basic general filtering algorithm
- Filters
  - Low pass
  - High pass
  - Band stop
  - Notch

Chapter 4: Enhancement in the Frequency Domain

- Laplacian in frequency domain
- High frequency emphasis
- Homomorphic filtering
- Sampling
- Convolution theorem
- Convolution with the DFT
- Masking/windowing vs. convolution
- Linear vs. circular convolution
- Length of result
- no wrap-around vs. wrap-around

Chapter 4: Enhancement in the Frequency Domain

- Zero padding
  - Why do we need it?
  - How much do we need?
- Separable convolution kernels
- Spectrum of a 2-d periodic signal
- Know how to find $H(u,v)$ from either $h(x,y)$ or a mask
- Know how to find magnitude & phase of the DFT
Chapter 5: Image Restoration

- Image degradation model:
  \[ g(x, y) = h(x, y) * f(x, y) + \eta(x, y) \]

- Degradation function:
  - Impulse response
  - Point spread function
  - Linear operator
    \[ H[af(x, y) + bf(x, y)] = aH[f(x, y)] + bH[f(x, y)] \]
  - Position invariant
    \[ g(x, y) = H(f(x, y)) \Rightarrow g(x - \alpha, y - \beta) = H(f(x - \alpha, y - \beta)) \forall \alpha, \beta \in \mathbb{R} \]
  - Linear position invariant impulse response
    \[ h(x, y) = H[\delta(x, y)] \]

Chapter 5: Noise-Only Filtering

- Many different types of averaging:
  - Arithmetic mean – sum and divide by \( mn \)
  - Geometric mean – multiply and take \( mn \)th root
  - Harmonic mean
  - Contra-harmonic mean
  - Median – keep middle value
  - Min and max
  - Midpoint
  - Alpha-trimmed mean – throw away outliers

- Adaptive filter:
  \[ \hat{j}(x, y) = \frac{\sigma^2_j}{\sigma^2_y}(g(x, y) - m_j) \]

Chapter 5: Estimating the Degradation Function

- Spatial domain & frequency domain
  \[ H(u, v) = \frac{G(u, v)}{F(u, v)} \]

- Modeling:
  - Atmospheric turbulence
  - Lens defocus
  - Camera motion
  - Inverse filtering
  - Pseudo-inverse filter
  - Wiener filter

- Need stochastic models of noise and image
- Need to know the degradation function

\[ \tilde{F}(u, v) = \left| \frac{S_r(u, v)H^*(u, v)}{S_f(u, v) |H(u, v)|^2 + S_r(u, v)} \right| G(u, v) \]
Chapter 5: Constrained Least Squares

- Convolution as matrix multiplication
  - Toeplitz matrix
  - Circulant matrix
  - Doubly block circulant matrix
  - Theory & practical limitations
- Constrained least squares
  - Minimization of criterion function
    \[ C = |P\hat{y}|^2 \]
  - Subject to a constraint
    \[ |x - h\hat{y}|^2 - |a|^2 = 0 \]
- Geometric transformations
  - Bilinear interpolation
- Blind deconvolution
  - Motion blur
  - Iterative blind deconvolution

Chapter 6: Color Image Processing

- Primary & secondary colors; light & pigment
  - 3 types of cones
  - Physiology, not physics
- Color fundamentals: brightness, hue, saturation, chromaticity, etc.
- Color models:
  - RGB – red, green, blue
  - CMY – cyan, magenta, yellow
  - CMYK – cyan, magenta, yellow, black
  - HSI – hue, saturation, intensity
  - RGB color cube

Chapter 6: Color Image Processing

- Pseudo-color image processing
- Multi-spectral images
- Full-color image processing
  - Histogram processing
  - Smoothing & sharpening
  - Segmentation
  - Edge detection

Chapter 9: Morphological Image Processing

- Set theory
- Dilation \[ A \oplus B = \{ z | (\hat{B}) \cap A \neq \emptyset \} \]
- Erosion \[ A \ominus B = \{ z | (\hat{B}) \subseteq A \} \]
- Opening \[ A \circ B = (A \ominus B) \oplus B \]
- Closing \[ A \bullet B = (A \oplus B) \ominus B \]
- Hit-or-miss transformation \[ A \ominus B = (A \ominus B_1) \cap (A' \ominus B_2) \]
- Thinning \[ A \ominus B = A \ominus (A \ominus B) \]
- Thickening \[ A \oplus B = A \cup (A \oplus B) \]
- Sequential thinning & thickening
  \[ A \ominus B = ((\ldots ((A \ominus B) \ominus B') \ldots) \ominus B) \]
Chapter 9: Morphological Algorithms

- Boundary extraction
- Region filling
- Extraction of connected components
- Homotopic transformations
  - Homotopic tree
- Skeletons
  - Medial axis transformation (MAT) – examples
  - Homotopic substitute for the skeleton – sequential thinning

Chapter 9: Morphological Operations on Gray Scale Images

- Morphology of 3-d surface
- Defined differently:
  - Complement: \( f'(x, y) = f(x, y) \)
  - Dilation: \( f \circ h = \max \left\{ f(x-s, y-t) + h(x, y) \mid (s, t) \in D_f, (x, y) \in D_h \right\} \)
  - Erosion: \( f \bullet h = \min \left\{ f(x+s, y+t) - h(x, y) \mid (s, t) \in D_f, (x, y) \in D_h \right\} \)
- Algorithms:
  - Smoothing
  - Gradient
  - Top-hat transformation

Chapter 10: Image Segmentation

- Segmentation subdivides an image into its constituent regions or objects.
- Edges, crack edges, edge direction
- Edge detection
- Gradient methods
  - Gradient operators
  - Vector gradient, scalar gradient
  - Edge strength & direction
- Laplacian methods
  - Laplacian operators
  - Zero crossing
  - Laplacian of Gaussian – Mexican hat function
- Edge linking

Chapter 10: Hough Transform

- Straight line detector
  - Straight lines translate to points
  - Points translate to sinusoidal curves
  - Parameterization: \( x \cos \theta + y \sin \theta = \rho \)
- Hough transform for circles
  - Parameterization: \( (x-x_c)^2 + (y-y_c)^2 = r^2 \)
- Generalized Hough transform:
  - R-table
  - \( A(x, y, \tau) \) data structure
  - Parameterization: \( x' = x + r(\phi) S \cos(\alpha(\phi) + \tau) \)
  \( y' = y + r(\phi) S \sin(\alpha(\phi) + \tau) \)
Chapter 10: Optimal Edges by Graph Search

- Graph theory
  - Root node & goal node
  - Parent node & child node
  - Edges
- Boundaries using graphs
  - Cost function:
    \[
    \text{cost} = H(p-q)
    \]
  - Minimum cost path
  - Graph search
- Heuristics:
  \[
  r(n) = g(n) + h(n)
  \]

Chapter 10: Thresholding

- Global thresholding – strengths and limitations
- Iterative algorithm for determining the threshold
- Adaptive thresholding
- Combination edge detection and thresholding

Chapter 10: Region Based Segmentation

- Divide image into disjoint sets:
  - \( H(R) = \text{TRUE} \)
  - \( H(R \cup R') = \text{FALSE} \)
- Region merging algorithm:
  1. Perform initial segmentation of image into many small regions.
  2. Choose any pair of adjacent regions, \( R_p \) and \( R_q \).
  3. Merge \( R_p \) and \( R_q \) if \( H(R_p \cup R_q) = \text{TRUE} \)
  4. Repeat 2 and 3 until no more regions can be merged.
- Region splitting algorithm
  - Region Splitting example
  - Quadtree representation
- Region split and merge

Chapter 11: Shape Representation and Description

- Chain codes and shape numbers
- Polygonal approximations
- Signatures
- Skeletons
  - Undirected graph
  - Adjacency matrix
  - Isomorphic graphs
- Diameter
- Major, minor axis
- Basic rectangle
- Eccentricity
Chapter 11: Shape Representation and Description

- Fourier descriptors
  - Invariance with respect to rotation, scaling, translation, starting point
- Compactness
- Euler number
- Texture
  - Histogram based approaches
  - Co-occurrence matrices
- 2d moments