Hit-or-miss Transformation

- Easier method of computing the hit-or-miss transformation:
  - Combine the two structuring elements, $B_1$ and $B_2$, into one template.
  - Move the template around in the image until you find a place where $B_1$ fits entirely in $A$, and $B_2$ fits entirely outside $A$.
  - The pixel that lies under the common origin of $B_1$ and $B_2$ belongs to the hit-or-miss transformation.

Boundary Extraction

- To get the boundary of $A$, we remove the erosion of $A$ from $A$.
- Characteristics of the boundary depend upon the structuring element.
Region Filling

Algorithm:
- Initialize $X_0$ to a point within the region to be filled.
- Dilation – the “paint” poured in at $X_0$ spreads out.
- Intersection – prevents the “paint” from “leaking” past the borders of the region.
- Algorithm terminates when $X_k$ does not change.

Must know in advance:
- $A$ is a valid (connected) border.
- $X_0$ is inside the border.

\[ X_k = (X_{k-1} \ominus B) \cap A \]

Filled set = $A \cup X_k$

Extraction of Connected Components

Algorithm:
- “Extraction of Unconnected Components” might be a better name for this algorithm
- Set $A$ may contain several components that are not connected to each other.
- Algorithm:  
  - Initialize $X_0$ to any point within $A$.
  - Algorithm terminates when $X_k$ does not change.
- Once one component of $A$ is identified, we can remove it from $A$ and search for other components.

\[ X_k = (X_{k-1} \ominus B) \cap A \]

Component = $X_k$

Thinning

\[ A \otimes B = A \rightarrow (A \ominus B) \]

- Similar to the boundary extraction algorithm, but with the hit-or-miss transformation substituted for erosion.
- Typically used to remove edge pixels from a set while leaving corner pixels intact.

Sequential Thinning

- Sequential thinning – repeated thinning using a sequence of structuring elements.

\[ \{B\} = \{B^1, B^2, B^3, \ldots, B^N\} \]

\[ A \otimes \{B\} = ((\cdots ((A \ominus B^1) \ominus B^2) \cdots) \ominus B^N) \]
Thinning Example

Set $A$ is the black area.

Original

1 iteration

2 iterations

5 iterations

Thickening

$A \odot B = A \cup (A \ominus B)$

- Dual operation of thinning; thickening is equivalent to thinning the complement (background)
- Like thinning except:
  - Union is substituted for intersection
  - The $B_1$ and $B_2$ components of the structuring elements are interchanged.
- Sequential thickening — repeated thickening using a sequence of structuring elements.

$\{B\} = \{B^1, B^2, B^3, \ldots, B^N\}$

$A \odot \{B\} = \{(\cdots ((A \odot B^1) \odot B^2) \cdots) \odot B^N\}$

Thickening Example

Set $A$ is the white area.

Original

1 iteration

2 iterations

5 iterations
Thickening Example

Set $A$ is the white area.

Original

10 iterations

25 iterations

Skeletons

- “Stick figure” representation of shape.
  - Used to recognize shapes.
  - Count the number and placement of branches and closed paths.
  - Invariant with respect to 2-d rotations.
  - Somewhat robust with respect to viewing angle.
- Medial axis transformation (MAT) – the MAT of a region $R$ with border $B$ is defined as follows: For each point $p$ in $R$, find its closest neighbor in $B$. If $p$ has more than one such neighbor, then it belongs to the medial axis (skeleton).
- Translation:
  - Fit circular disks into set $R$.
  - For any disk that touches the border of $R$ in more than one place, its center point is part of the skeleton of $R$.

Homotopic Transformations

- A transformation is **homotopic** if it does not change the contiguity relation between regions and holes.
- A transformation is homotopic if it does not change the homotopic tree.
- Thinning and thickening are homotopic transformations.
- Sequential thinning converges to the homotopic substitute for the skeleton.
Skeletonizing Algorithms

- The skeleton of a continuous set is clearly defined and simple to construct.
- For a discrete set (pixels), the skeleton is more difficult to define.
  - How is the border defined?
  - How is distance defined?
- Sequential thinning converges to the homotopic substitute for the skeleton.
- An algorithm for the morphological skeleton is given in the text, but the morphological skeleton may not be connected.
- Another skeletonizing algorithm, not based on morphological operations, is given in section 11.1.5.

Morphological Operations

- Dilation
- Erosion
- Opening
- Closing
- Hit-or-miss transformation
- Thinning
- Thickening

Morphological Algorithms

- Bridging Gaps
- Elimination of irrelevant detail
- Noise suppression
- Shape matching
- Boundary extraction
- Region filling
- Extraction of connected components
- Homotopic skeleton