STATIC TOMOGRAPHIC RECONSTRUCTION OF THE TIME VARYING IONOSPHERE

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ABSTRACT

An image of electron density in a vertical slice through the ionosphere can be created using tomographic techniques. An ionospheric tomography system consists of a satellite and several ground stations. Ionospheric electron density can change significantly during the time it takes to collect enough data for one image. In many other applications of tomography, motion of an object in the source image produces obvious artifacts in the reconstruction such as streaking along object edges. For ionospheric tomography, the effects of motion in the source image are less obvious but nevertheless cause serious errors in the reconstruction. This paper discusses the effects of motion in the ionosphere on static ionospheric tomography reconstructions, and suggests ways to detect that a reconstruction has been corrupted by motion in the source image. Several simulated examples are presented using two different ionospheric tomography reconstruction algorithms.

1. INTRODUCTION

Ionospheric tomography (IT) is a technique that is used to reconstruct images of ionospheric electron density [1, 2]. An IT system consists of a satellite in polar orbit and several ground stations located along a line of longitude underneath the orbit of the satellite. Using the differential Doppler technique, total electron content (TEC) data are obtained along paths between the satellite and the ground stations. TEC is the integral of electron density, so tomographic techniques are used to reconstruct an image of ionospheric electron density in a vertical plane that includes the satellite orbit and the ground stations. The principal difficulty associated with IT is that the TEC data do not form a complete set of projections of the source image [3, 4]. Therefore all practical IT algorithms use a priori information to supplement the information contained in the TEC data.

Another source of information that can be used to supplement TEC data is ionosonde data. An ionosonde is essentially a one dimensional radar looking straight up. The ionosonde gives electron density values in a vertical line above the location of the ionosonde up to the altitude of the maximum electron density. The ionosonde only gives information on the bottom side of the ionosphere and only over a very limited area.

One of the sources of errors for IT systems is movement in the ionosphere during data collection. A typical IT system tracks a satellite through about 70° of latitude. A satellite at an altitude of 1000 km takes approximately 20 minutes to traverse 70°. Significant motion in the ionosphere can occur during the 20 minutes required to collect data for a single IT reconstruction. The currently accepted practice in IT is to assume that all data are collected simultaneously and to perform the reconstruction as if the source image were not varying [3, 4]. However, movement in the ionosphere the effecting motion in the is a significant contributor to errors in IT reconstructions. This paper will examine and characterize the errors that occur in static IT reconstructions due to movement in the ionosphere.

2. IONOSPHERIC TOMOGRAPHY ALGORITHMS

Ionospheric tomography algorithms can be divided into two classes: pixel based methods and nonpixel based methods. Since different algorithms respond to time variations in the ionosphere in different ways, this paper will present results from both a pixel based algorithm and a nonpixel based algorithm. Furthermore, since ionosonde information is often used to supplement TEC data, this paper will also present results using ionosonde information.

The pixel based algorithm that will be used to demonstrate the effects of motion in the source image is smoothness and conjugate gradients (SCG) [5]. The SCG algorithm combines smoothness constraints similar to those proposed by Felmers [6] with solution by conjugate gradients.

It is known that ionospheric electron density tends to be constant in the horizontal direction, smooth in the vertical direction, and very small at low and high altitudes. These characteristics are incorporated into the SCG algorithm by minimizing the first derivative in the horizontal direction, the second derivative in the vertical direction, and the value of the pixels at the bottom and top of the reconstruction. This leads to a large, sparse system of equations that can be solved using any convenient least squares technique. The method of conjugate gradients will be used here, since its convergence is not affected by inconsistent data.

The nonpixel based algorithm that will be used to demonstrate the effects of motion in the source image is the residual correction method (RCM) [7]. The image $g(\theta, r)$ is expressed as the weighted sum of orthonormal basis images.
\[ g(\theta, r) = \sum_{i=1}^{K} x_i \phi_i(\theta, r), \] (1)

where the weights \( \{ x_1, \ldots, x_K \} \) are unknown. The set of basis images \( \{ \phi_1, \ldots, \phi_K \} \) is separable; the Fourier basis is used in the horizontal direction, and empirical orthonormal basis functions are used in the vertical direction.

Even with empirical orthonormal functions for the vertical basis, the reconstruction is generally not uniquely determined, so the reconstruction problem is partitioned into a set of smaller problems, each of which possesses a unique solution. The RCM algorithm then iteratively cycles through the partitions of the problem. RCM does not guarantee nonnegativity of the solution, but if the data are consistent with the \textit{a priori} information, then RCM tends to produce a nonnegative solution.

### 3. ARTIFACTS DUE TO MOTION

In other applications of tomography where complete projection data are available, imaging artifacts from movement of an object during data collection include smearing and streaking. Even very little motion can produce obvious artifacts in the reconstruction. Since ionospheric electron density is a smooth function, and the projection data for an IT system are incomplete, some of the artifacts produced by movement in the ionosphere during data collection are more difficult to identify than the artifacts produced in other applications of tomography. In general, significant motion in the ionosphere can produce distortions in the reconstruction that resemble valid ionospheric features.

For any TEC data set, no matter how corrupted by time variations in the ionosphere, there always exists a source image that produces the observed data. This is particularly true for IT because the projection data are incomplete. Therefore the TEC data cannot be inconsistent with itself; however, it may be inconsistent with \textit{a priori} assumptions. For example, the reconstructed image could contain features that are clearly impossible, such as negative electron densities.

For IT the types of errors that can be caused by movement of the ionosphere during data collection can be summarized as follows: (1) clearly visible artifacts, e.g., streaking; (2) increased magnitude of residual; (3) negative pixels in reconstruction; (4) geometric distortion, e.g., features displaced from actual position. The presence of clearly visible artifacts can be used to detect images that are corrupted by motion in the source image, but these are not often visible in IT reconstructions. The magnitude of the residual and number of negative pixels can, under some conditions be used to detect corrupted images. Geometric distortion is a serious error that cannot be used to detect corrupted images.

The effects of movement in the ionosphere during data collection will be evaluated in terms of the four categories stated above. Both SCG and RCM will be evaluated. Also, SCG with ionosonde data in addition to TEC data will be evaluated to determine the effect of additional information on the reconstruction errors.

### 4. SIMULATIONS

The source image used in the simulations presented in this paper is shown in Figure 1a. TEC data was simulated for this image using 13 receivers from \(-30^\circ\) to \(30^\circ\) latitude in increments of \(5^\circ\) and satellite positions from \(-35^\circ\) to \(35^\circ\) latitude in increments of \(0.25^\circ\). In the simulations where ionosonde data was used, data was simulated for 7 ionosondes from \(-30^\circ\) to \(30^\circ\) latitude in increments of \(10^\circ\).

![Figure 1: Original image and reconstructions using static data. (a) Original image. (b) Reconstruction using SCG. (c) Reconstruction using SCG with ionosonde information. (d) Reconstruction using RCM.](image)

The reconstructions of Figure 1 show how each of the algorithms behave when the source image is not moving. The reconstruction in Figure 1b was performed using the SCG algorithm. The reconstruction is smeared in the vertical direction, because the SCG algorithm uses minimal \textit{a priori} information. However, the three peaks are reconstructed at the correct latitude and altitude.

The reconstruction in Figure 1c was performed using the SCG algorithm with ionosonde information. Since an ionosonde is located directly under the central peak, there is a significant improvement in vertical resolution for the central peak though not for the outside peaks. It is interesting to note that the resolution is increased on the top side of the central peak even though the ionosonde provides information only about the bottom side. Information about the bottom side profile is sufficient, when combined with the TEC data, to increase the resolution on both the bottom side and top side.

The reconstruction in Figure 1d was performed using the RCM algorithm. The vertical resolution is much greater for the RCM algorithm, because much stronger use is made of \textit{a priori} information. The \textit{a priori} information consisted of a set of 98 profiles with the altitude of the peak electron density ranging from 250 km to 550 km.

All three of the reconstructions of Figure 1 are plot-
ted using 98 pixels in the horizontal direction and 25 pixels in the vertical direction. For the SCG algorithm, this means that the reconstruction was performed using 2450 basis functions, since for the SCG algorithm each pixel is a basis function. For the RCM algorithm, 3 vertical basis functions and 21 horizontal basis function where used, for a total of 63 basis images.

Figure 2: Original image and reconstructions where the electron density drifted to the left during data collection: (a) Original image, (b) Reconstruction using SCG, (c) Reconstruction using SCG with ionosonde information, (d) Reconstruction using RCM.

Figure 2 shows the effect of the electron density drifting to the left, in the opposite direction from the satellite motion, during data collection. The source image was allowed to drift a total of 10° from 5° left of center to 5° right of center. The source image is shown again in Figure 2a. A reconstruction using SCG with no ionosonde information is shown in Figure 2b. The outer peaks are displaced toward the center of the image, because when the outer peaks drift in the opposite direction from the satellite motion, they are located closer to the center at the time the satellite is directly overhead. All three peaks are reconstructed at a slightly higher altitude than in the source image or in the reconstruction of Figure 1b.

A reconstruction using SCG with ionosonde information is shown in Figure 2c. Clearly, motion in the ionosphere may be detected by examining how the ionosonde measurements evolve over time; however, for a static reconstruction only one set of ionosonde measurements can be included. The ionosonde measurements were taken when the satellite was at 0° latitude. The central peak is forced to the correct altitude by the data from the ionosonde located at 0°, but the outer peaks are displaced upwards even more in Figure 2b. Also, there is an artifact at the top center of the reconstruction of Figure 2c.

A reconstruction using RCM is shown in Figure 2d. The background electron density is reconstructed at the correct altitude, but the three peaks are all displaced upwards as in Figure 2b. The distortions at the left and right edges are not significant, since there is no TEC data for those areas of the image.

Figure 3: Original image and reconstructions where the electron density drifted to the right during data collection: (a) Original image, (b) Reconstruction using SCG, (c) Reconstruction using SCG with ionosonde information, (d) Reconstruction using RCM.

Figure 3 shows the effect of the electron density drifting to the right, in the same direction as the satellite motion, during data collection. The source image was allowed to drift a total of 10° from 5° right of center to 5° left of center. The source image is shown again in Figure 3a. A reconstruction using SCG with no ionosonde information is shown in Figure 3b. The outer peaks are displaced away from the center of the image, and all three peaks are reconstructed at a slightly higher altitude than in the source image or in the reconstruction of Figure 1b. A reconstruction using SCG with ionosonde information is shown in Figure 3c. The central peak is forced to the correct altitude by the ionosonde information, but the outer peaks are badly distorted, and there is an artifact at the top center of the image. A reconstruction using RCM is shown in Figure 3d where all three peaks are displaced downwards.

Most of the reconstructions do not show any clearly visible artifacts such as smearing or streaking. Figures 2c and 3c show very minor artifacts, probably due to the conflict between the TEC data and the ionosonde data.

Table 1 shows the magnitude of the residual vector for all 9 reconstructions as calculated for the TEC measurements only. The residual vector is the difference between the observed TEC data and TEC data calculated from the reconstruction. The SCG algorithm does not show any increase in the residual when the source image is moving. The large number of pixels and small amount of a priori information gives the SCG algorithm enough degrees of freedom to calculate a reconstruction to match almost any TEC data.
When ionosonde information is included in the data for the SCG algorithm, there is a small increase in the residual when the source image is moving, but not enough to reliably determine that there is a problem. For the RCM algorithm, since it uses the strongest a priori information, there is a dramatic increase in the residual when the source image is moving. In order to identify reconstructions that are corrupted by movement in the ionosphere, stronger use of a priori information is needed.

Table 1: Length of residual vector in electrons/m².

<table>
<thead>
<tr>
<th></th>
<th>Static</th>
<th>Mov. Left</th>
<th>Mov. Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCG</td>
<td>779</td>
<td>7568</td>
<td>7219</td>
</tr>
<tr>
<td>SCG w/Iono.</td>
<td>2798</td>
<td>3881</td>
<td>3034</td>
</tr>
<tr>
<td>RCM</td>
<td>1997</td>
<td>6000</td>
<td>27191</td>
</tr>
</tbody>
</table>

Table 2: Number of negative pixels.

<table>
<thead>
<tr>
<th></th>
<th>Static</th>
<th>Mov. Left</th>
<th>Mov. Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCG</td>
<td>12</td>
<td>49</td>
<td>0</td>
</tr>
<tr>
<td>SCG w/Iono.</td>
<td>956</td>
<td>1040</td>
<td>986</td>
</tr>
<tr>
<td>RCM</td>
<td>0</td>
<td>216</td>
<td>208</td>
</tr>
</tbody>
</table>

Table 2 shows the number of negative pixels for all 9 reconstructions. For the SCG algorithm there is no consistent increase in the number of negative pixels when the source image is moving, and when ionosonde information is used there is only a small increase in the number of negative pixels. For the RCM algorithm there is a much larger increase in the number of negative pixels when the source image is moving. Again, stronger use of a priori information is needed to identify reconstructions that are corrupted by movement in the ionosphere.

Figure 4: Diagram showing how moving ionospheric disturbances are displaced in altitude. The initial and final positions of an ionospheric disturbance are shown in solid lines, and the apparent position is shown in dotted lines.

All of the reconstructions of Figures 2 and 3 show geometric distortion of the three disturbances. The horizontal displacement of the disturbances is not actually a distortion, since the horizontal position is correct for the time when the satellite is over the region where the disturbance is found. The vertical displacement of the disturbances is a more serious problem. Figure 4 illustrates why this vertical displacement occurs. When the satellite is at position 1, the data for ground station B shows the peak of the disturbance. When the satellite has moved to position 2, the disturbance has moved so that the data for ground station A shows the peak of the disturbance. When the algorithm attempts to find a fixed position for the disturbance, it locates the disturbance at the intersection of the lines 1B and 2A. Therefore the horizontal movement of the disturbance during data collection causes a vertical displacement in the reconstruction of the disturbance.

5. CONCLUSION

Time variations in the source image are a significant contributor to the errors in static reconstructions of the ionosphere. These errors include artifacts and geometric distortion. Errors due to motion in the ionosphere during data collection are serious, since geometric distortion cannot be detected from examination of a single IT reconstruction, and artifacts are difficult to identify and not always present. The TEC data cannot be inconsistent with itself, but it can be inconsistent with a priori information. Therefore, it is possible to detect that a reconstruction is distorted due to motion in the source image if the reconstruction algorithm uses sufficiently strong a priori information. This can be accomplished by analysis of the magnitude of the residual and the number of negative pixels. Ionosonde information can be useful in detecting motion; however, inclusion of a single set of ionosonde scans in the data set can lead to additional distortion in the reconstruction and is generally not sufficient to detect motion.

Acknowledgments. This material is based upon work supported by the Office of Naval Research under grant N00014-95-1-0850.

6. REFERENCES