

## Exam 2 Review

- One hour to complete
- Closed book, closed notes, no calculator necessary
- Two 8.5 by 11 sheets with anything you want on it:
  - You might want to make one sheet the one from the first exam
  - You will be allowed three sheets for the final
- Covers chapters 4, 5, 6, 9, 10, 11. (Chapter 8 will be on the final)
- Six questions

## Chapter 4: Enhancement in the Frequency Domain

- Properties of the DFT
  - Symmetry properties  $F(u, v) = F^*(-u, -v)$
  - Periodicity  $F(u, v) = F(u+M, v+N)$
  - Centered & uncentered, frequency shifting
  - Spatial shifting
  - Scaling
  - Reciprocal relationship with spatial domain
- Basic general filtering algorithm
- Filters
  - Low pass
  - High pass
  - Band stop
  - Notch

## Chapter 4: Enhancement in the Frequency Domain

- Laplacian in frequency domain
- High frequency emphasis
- Homomorphic filtering
- Sampling  $\Delta u = \frac{f_{sx}}{M} = \frac{1}{M \Delta x}$   $\Delta v = \frac{f_{sy}}{N} = \frac{1}{N \Delta y}$
- Convolution theorem
  - Convolution with the DFT
  - Masking/windowing vs. convolution
- Linear vs. circular convolution
  - Length of result
  - no wrap-around vs. wrap-around

## Chapter 4: Enhancement in the Frequency Domain

- Zero padding
  - Why do we need it?
  - How much do we need?
- Separable convolution kernels
- Spectrum of a 2-d periodic signal
- Know how to find  $H(u, v)$  from either  $h(x, y)$  or a mask
- Know how to find magnitude & phase of the DFT

## Chapter 5: Image Restoration

- Image degradation model:

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

- Degradation function:

- Impulse response
- Point spread function
- Linear operator

$$H[af_1(x, y) + bf_2(x, y)] = aH[f_1(x, y)] + bH[f_2(x, y)]$$

- Position invariant

$$g(x, y) = H(f(x, y)) \Rightarrow g(x - \alpha, y - \beta) = H(f(x - \alpha, y - \beta)) \quad \forall \alpha, \beta \in \mathbf{R}$$

- Linear position invariant impulse response

$$h(x, y) = H[\delta(x, y)]$$

## Chapter 5: Noise-Only Filtering

- Many different types of averaging:
  - Arithmetic mean – sum and divide by  $mn$
  - Geometric mean – multiply and take  $mn^{\text{th}}$  root
  - Harmonic mean
  - Contra-harmonic mean
  - Median – keep middle value
  - Min and max
  - Midpoint
  - Alpha-trimmed mean – throw away outliers
- Adaptive filter:

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_n^2}{\sigma_L^2} (g(x, y) - m_L)$$

## Chapter 5: Noise-Only Filtering

- Periodic noise reduction: Band reject filters, notch filters; ideal, Butterworth, Gaussian
- Adaptive notch filter:

$$\hat{f}(x) = g(x) - w(x)\eta(x)$$

$$\sigma^2(x) = \frac{1}{P} \sum_{s \in S} (\hat{f}(x+s) - \bar{\hat{f}}(x))^2$$

## Chapter 5: Estimating the Degradation Function

- Spatial domain & frequency domain  $H(u, v) = \frac{G(u, v)}{F(u, v)}$
- Modeling:
  - Atmospheric turbulence
  - Lens defocus
  - Camera motion
- Inverse filtering
- Pseudo-inverse filter
- Wiener filter
  - Need stochastic models of noise and image
  - Need to know the degradation function

$$\hat{F}(u, v) = \left( \frac{S_f(u, v) H^*(u, v)}{s_f(u, v) |H(u, v)|^2 + S_{\eta}(u, v)} \right) G(u, v)$$

## Chapter 5: Constrained Least Squares

- Convolution as matrix multiplication
  - Toeplitz matrix
  - Circulant matrix
  - Doubly block circulant matrix
  - Theory & practical limitations
- Constrained least squares
  - Minimization of criterion function
  - Subject to a constraint
- Geometric transformations
  - Bilinear interpolation
- Blind deconvolution
  - Motion blur
  - Iterative blind deconvolution

$$C = \|P \hat{f}\|^2$$

$$\|g - H \hat{f}\|^2 - \|\eta\|^2 = 0$$

## Chapter 6: Color Image Processing

- Primary & secondary colors; light & pigment
  - 3 types of cones
  - Physiology, not physics
- Color fundamentals: brightness, hue, saturation, chromaticity, etc.
- Color models:
  - RGB – red, green, blue
  - CMY – cyan, magenta, yellow
  - CMYK – cyan, magenta, yellow, black
  - HSI – hue, saturation, intensity
  - RGB color cube

## Chapter 6: Color Image Processing

- Pseudo-color image processing
- Multi-spectral images
- Full-color image processing
  - Histogram processing
  - Smoothing & sharpening
  - Segmentation
  - Edge detection

## Chapter 9: Morphological Image Processing

- Set theory
- Dilation  $A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$
- Erosion  $A \ominus B = \{z | (B)_z \subseteq A\}$
- Opening  $A \circ B = (A \ominus B) \oplus B$
- Closing  $A \bullet B = (A \oplus B) \ominus B$
- Hit-or-miss transformation  $A \otimes B = (A \ominus B_1) \cap (A^c \ominus B_2)$
- Thinning  $A \otimes B = A - (A \oplus B)$
- Thickening  $A \odot B = A \cup (A \oplus B)$
- Sequential thinning & thickening

$$A \otimes \{B\} = ((\dots((A \otimes B^1) \otimes B^2) \dots) \otimes B^N)$$

## Chapter 9: Morphological Algorithms

- Boundary extraction
- Region filling
- Extraction of connected components
- Homotopic transformations
  - Homotopic tree
- Skeletons
  - Medial axis transformation (MAT) – examples
  - Homotopic substitute for the skeleton – sequential thinning

## Chapter 9: Morphological Operations on Gray Scale Images

- Morphology of 3-d surface
- Defined differently:

- Complement:

$$f^c(x, y) = -f(x, y)$$

- Dilation:

$$f \oplus b = \max \{ f(x-s, y-t) + b(x, y) \mid (s-x, t-y) \in D_f, (x, y) \in D_b \}$$

- Erosion:

$$f \ominus b = \min \{ f(x+s, y+t) - b(x, y) \mid (s+x, t+y) \in D_f, (x, y) \in D_b \}$$

- Algorithms:
  - Smoothing
  - Gradient
  - Top-hat transformation

## Chapter 10: Image Segmentation

- Segmentation subdivides an image into its constituent regions or objects.
- Edges, crack edges, edge direction
- Edge detection
- Gradient methods
  - Gradient operators
  - Vector gradient, scalar gradient
  - Edge strength & direction
- Laplacian methods
  - Laplacian operators
  - Zero crossing
  - Laplacian of Gaussian – Mexican hat function
- Edge linking

## Chapter 10: Hough Transform

- Straight line detector

- Straight lines translate to points
- Points translate to sinusoidal curves

- Parameterization:

$$x \cos \theta + y \sin \theta = \rho$$

- Hough transform for circles

- Parameterization:  $(x-x_c)^2 + (y-y_c)^2 = r^2$

- Generalized Hough transform:

- R-table

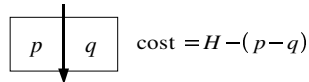
- A(x, y, τ) data structure

- Parameterization:  $x^R = x + r(\phi) S \cos(\alpha(\phi) + \tau)$

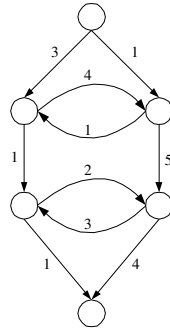
$$y^R = y + r(\phi) S \sin(\alpha(\phi) + \tau)$$

## Chapter 10: Optimal Edges by Graph Search

- Graph theory
  - Root node & goal node
  - Parent node & child node
  - Edges
- Boundaries using graphs
  - Cost function:



- Minimum cost path
- Graph search
- Heuristics:
  - $r(n) = g(n) + h(n)$



## Chapter 10: Thresholding

- Global thresholding – strengths and limitations
- Iterative algorithm for determining the threshold
- Adaptive thresholding
- Combination edge detection and thresholding

## Chapter 10: Region Based Segmentation

- Divide image into disjoint sets:
  - $H(R_i) = \text{TRUE}$
  - $H(R_i \cup R_j) = \text{FALSE}$
- Region merging algorithm:
  1. Perform initial segmentation of image into many small regions.
  2. Choose any pair of adjacent regions,  $R_p$  and  $R_q$ .
  3. Merge  $R_p$  and  $R_q$  if  $H(R_p \cup R_q) = \text{TRUE}$
  4. Repeat 2 and 3 until no more regions can be merged.
- Region splitting algorithm
  - Region Splitting example
  - Quadtree representation
- Region split and merge

## Chapter 11: Shape Representation and Description

- Chain codes and shape numbers
- Polygonal approximations
- Signatures
- Skeletons
  - Undirected graph
  - Adjacency matrix
  - Isomorphic graphs
- Diameter
- Major, minor axis
- Basic rectangle
- Eccentricity

## Chapter 11: Shape Representation and Description

- Fourier descriptors
  - Invariance with respect to rotation, scaling, translation, starting point
- Compactness
- Euler number
- Texture
  - Histogram based approaches
  - Co-occurrence matrices
- 2d moments