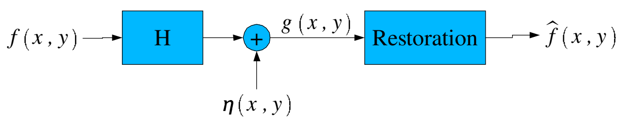
Image Degradation Model



$$g(x,y)=H[f(x,y)]+\eta(x,y)$$

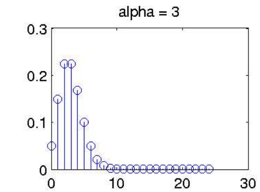
$$g(x,y)=h(x,y)*f(x,y)+\eta(x,y)$$

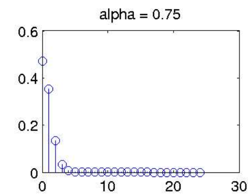
$$G(u,v)=H(u,v)F(u,v)+N(u,v)$$

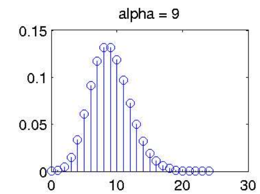
Noise Models

Poisson noise model:

$$p(k)=\frac{\alpha^k}{k!}e^{-\alpha}$$







Noise Models

- Gaussian

$$p(z)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(z-\mu)^2/2\sigma^2}$$

- Rayleigh

$$p(z)=\begin{cases}\frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z\geq a\\ 0 & \text{for } z<a\end{cases}$$

- Erlang (Gamma)

$$p(z)=\begin{cases}\frac{a^b z^{b-1}}{(b-1)!e^{-az}} & \text{for } z\geq 0\\ 0 & \text{for } z<0\end{cases}$$

Noise Models

- Exponential

$$p(z)=\begin{cases}ae^{-az} & \text{for } z\geq 0\\ 0 & \text{for } z<0\end{cases}$$

- Uniform

$$p(z)=\begin{cases}\frac{1}{(b-a)} & \text{for } a\leq z\leq b\\ 0 & \text{otherwise}\end{cases}$$

- Impulse (Salt & Pepper)

$$p(z)=\begin{cases}P_a & \text{for } z=a\\ P_b & \text{for } z=b\\ 0 & \text{otherwise}\end{cases}$$

Estimation of Noise Parameters

- What we really need is an image containing only noise
- Try one of the following:
  - Take a picture of a uniform gray background
  - For noise from a CCD: take a picture with the camera shutter closed
  - Use a uniform gray area in an existing image
- Then calculate mean and variance
- Also examine the histogram and fit one of the noise models

Noise-only Filtering

$$g(x,y)=f(x,y)+\eta(x,y)$$

- Assume f(x,y) doesn't change too much in a small region
- Use some form of averaging within the region
- How many different ways can we do an average?

Noise-only Filtering

- Arithmetic mean

$$\hat{f}(x,y)=\frac{1}{mn}\sum_{(s,t)\in S_{xy}} g(s,t)$$

- Geometric mean

$$\hat{f}(x,y)=\left[\prod_{(s,t)\in S_{xy}} g(s,t)\right]^{\frac{1}{mn}}$$

Noise-only Filtering

- Harmonic mean

$$\hat{f}(x,y)=\frac{mn}{\sum_{(s,t)\in S_{xy}}\frac{1}{g(s,t)}}$$

- Contra-harmonic mean

$$\hat{f}(x,y)=\frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Noise-only Filtering

- Median filter

$$\hat{f}(x,y) = \text{median}_{(s,t)\in S_{xy}}\{g(s,t)\}$$

- Min and max filters

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}}\{g(s,t)\} \qquad \hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\{g(s,t)\}$$

- Midpoint filter

$$\hat{f}(x,y) = \frac{1}{2}\left\{\min_{(s,t)\in S_{xy}}\{g(s,t)\} + \max_{(s,t)\in S_{xy}}\{g(s,t)\}\right\}$$

- Alpha-trimmed mean filter – throw out some percent of largest and smallest values, then calculate mean

Adaptive Filters

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left(g(x,y) - m_L\right)$$

Adaptive Median Filter

- Vary the size of the window over which the median is calculated
- Useful when there is very heavy impulse noise
- Basic idea – if the median is equal to either the max or min value, then the median is judged to be not very good, and the window is enlarged until a good median is obtained

Periodic Noise Reduction: Band Reject Filters

- "Ideal"

$$H(u,v)=\begin{cases}0 & \text{if } D_0-\frac{W}{2}\le D(u,v)\le D_0+\frac{W}{2}\\ 1 & \text{otherwise}\end{cases}$$

- Butterworth

$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}}$$

- Gaussian

$$H(u,v)=1-\exp\left[-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right]^2\right]$$

Notch Filters

- Not circularly symmetric filters
- Must be careful to satisfy symmetry conditions

$$D_1(u,v)=\left[(u-(M/2)-u_0)^2+(v-(N/2)-v_0)^2\right]^{1/2}$$

$$D_2(u,v)=\left[(u-(M/2)+u_0)^2+(v-(N/2)+v_0)^2\right]^{1/2}$$

$$H(u,v)=\begin{cases}0 & \text{if } D_1(u,v)\le D_0 \text{ or } D_2(u,v)\le D_0\\ 1 & \text{otherwise}\end{cases}$$

Optimum (Adaptive) Notch Filter

- Suppose we have a signal corrupted by a quasi-periodic interference with amplitude that varies slowly over time
- The amplitude variations "smear" the spikes in the frequency domain
- Using a wider notch filter will filter out too much of the signal
- Solution:
  - Extract the basic periodic interference signal with a notch filter
  - Do a local curve fit to find the local amplitude of the interference
  - Subtract the locally weighted interference from the signal

Optimum (Adaptive) Notch Filter

$$\hat{f}(x) = g(x) - w(x)\eta(x)$$

$$\sigma^2(x) = \frac{1}{P}\sum_{s \in S}\left(\hat{f}(x+s) - \overline{\hat{f}}(x)\right)^2$$

$$\sigma^2(x) = \frac{1}{P}\sum_{s \in S}\left(g(x+s) - w(x+s)\eta(x+s) - \overline{(g(x) - w(x)\eta(x))}\right)^2$$

$$\sigma^2(x) = \frac{1}{P}\sum_{s \in S}\left(g(x+s) - \overline{g}(x) - w(x)(\eta(x+s) - \overline{\eta}(x))\right)^2$$

Optimum (Adaptive) Notch Filter

$$\sigma^2(x) = \frac{1}{P}\sum_{s \in S}\left(g(x+s) - \overline{g}(x) - w(x)(\eta(x+s) - \overline{\eta}(x))\right)^2$$

$$\frac{\partial \sigma^2(x)}{\partial w(x)} = \frac{1}{P}\sum_{s \in S} 2\left(g(x+s) - \overline{g}(x) - w(x)(\eta(x+s) - \overline{\eta}(x))\right)(\eta(x+s) - \overline{\eta}(x)) = 0$$

$$\frac{1}{P}\sum_{s \in S} g(x+s) = \overline{g}(x) \qquad \frac{1}{P}\sum_{s \in S} \eta(x+s) = \overline{\eta}(x)$$

$$\frac{1}{P}\sum_{s \in S} g(x)\eta(x+s) = \overline{g(x)\eta(x)} \qquad \frac{1}{P}\sum_{s \in S} \eta^2(x+s) = \overline{\eta^2(x)}$$

Optimum (Adaptive) Notch Filter

$$\frac{\partial \sigma^2(x)}{\partial w(x)} = 2\left(\overline{g(x)\eta(x)} - \overline{g}(x)\overline{\eta}(x) - w(x)\left(\overline{\eta^2(x)} - \left(\overline{\eta}(x)\right)^2\right)\right) = 0$$

$$w(x) = \frac{\overline{g(x)\eta(x)} - \overline{g}(x)\overline{\eta}(x)}{\overline{\eta^2(x)} - \left(\overline{\eta}(x)\right)^2}$$